Integer Programming Formulations for Compact Single Round Robin Tournaments

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Abstract. We consider the problem of finding an optimal schedule for compact single round robin tournaments. To this end, we discuss one polynomial size and two exponential size integer programming formulations of this problem. We compare the strength of the linear programming relaxations of these three models. Moreover, we show that the pricing problems of both exponential size formulations can be solved in polynomial time.

Keywords: Sport scheduling · Round Robin · Mixed Integer Programming · Branch and Price

Integer programming continues to be a very popular way to obtain a schedule for a round robin tournament. It does not only allow to automatically generate schedules, but also to easily incorporate different kinds of constraints to find a schedule addressing needs of a specific tournament. To substantiate this claim of widespread use of integer programming, it is a fact that the literature contains lots of papers demonstrating the use of integer programming for finding schedules in sports timetabling. Well-known surveys are by Rasmussen and Trick [10] and Kendall et al. [8]. Complexity results regarding round robin tournaments are provided by Easton [6], Briskorn et al. [3], and Van Bulck and Goossens [12]. Integer programming formulations have also been studied, among others, by Trick [11] and Briskorn and Drexl [2]. More recently, the international timetabling competition [4] featured a round robin sports timetabling problem, and most of the submissions used integer programming in some way to obtain a good schedule. Other recent contributions using integer programming for sports timetabling include Durán et al. [5] and Bouzarth et al. [1]. For more papers in the field of sports scheduling, we refer to Knust [9], who maintains an elaborate classification of literature on sports scheduling.

We aim to take a fresh look at the problem of finding an optimal schedule for compact single round robin tournaments using integer programming techniques. Given a set of n teams, a *compact single round robin tournament* consists of $n-1$ rounds of matches such that each team plays against exactly one other team per round and each pair of opponents meets exactly once. The FIFA World Cup group stage is an example of a compact single round robin tournament. This type is also common for chess tournaments, for instance for all important tournaments between London 1862 and Curaçao Candidates tournaments of 1962 [7]. In the following, we assume that each hypothetical match at a specific round has an associated cost. Our goal is then to find a schedule that minimizes total cost. By finding cost-minimizing optimal schedules for compact single round robin tournaments, one can take practical considerations and wishes into account.

We discuss three different integer programming formulations for finding an optimal schedule. First, we consider a *traditional formulation* using $O(n^3)$ many variables and constraints that is also studied by Trick [11] and Briskorn and Drexl [2]. This model introduces, for every hypothetical match between two teams and round, a binary decision variable that indicates whether this match is scheduled on that round. We compare this formulation with two alternative novel formulations that are based on different encodings of a schedule: a matching and a permutation formulation.

The *matching formulation* introduces a binary variable for each pair (M, r), where M is a perfect matching of the n teams and r is the index of a round. These variables encode the entire schedule of round r. Instead of fixing the schedule of a round, the *permutation formulation* fixes, for a given team, the order of matches that it plays against all other teams. That is, it introduces a variable for each team t and each permutation of $\{1, \ldots, n\} \setminus \{t\}$. Note that, in contrast to the traditional formulation, both the matching and permutation formulation have exponentially many variables.

Our main contributions are twofold:

1. Despite the exponential number of variables, we show that the linear programming relaxations of both the matching and permutation formulation can be solved in polynomial time. To this end, we show that the pricing problems of both formulations reduce to finding perfect matchings in suitably defined auxiliary graphs.
2. We provide a comparison of the optimal values of linear programming relaxations of the three different models. We show that the traditional and permutation formulation both provide the same optimal value of their linear programming relaxations, whereas the matching formulation is at least as strong as the other two formulations. In particular, we show that it can be strictly stronger.

Moreover, we discuss how the matching and permutation formulation can be solved within a branch-and-price framework.

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