Scheduling products at paced assembly lines with a multiple-piece flow

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1 Introduction

In the paper, we study product scheduling at paced assembly lines with a multiple-piece flow. Due to increasing customization of the products, the conventional paced assembly lines with one-piece flow manufacturing come to its limits. Since individual workpieces may have very different features and, therefore, require different processing times at the same station, the so-called idle times grow, unit production costs increase and outweigh the advantages of learning effects, of specialization, and of organizational transparency due to pacing. Therefore, modern firms look for innovative manufacturing concepts in order to produce customized products at low costs. One of such concepts is multiple-piece flow manufacturing. According to this concept, several workpieces may be launched simultaneously as a bundle to pass several sequentially arranged stations. At each station, a bundle of products is simultaneously available for processing for a given amount of time, called *cycle time*. Afterward, it is moved to the next station with some transportation equipment, such as a conveyor belt or automatically steered skids. Cycle times at multiple-piece

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flow assembly lines are usually larger than that of one-piece flow assembly lines. Multiple-piece flow is more suited to manufacturing of customized products, since the variation of processing times between cycles and stations can be smoothed by combining several workpieces in one bundle. We have observed multiple-piece flow manufacturing at several medium-sized enterprises, such as at productions of soldering machines and of industrial cranes.

Our focus is on product scheduling. Given a set of customized products $j \in J$ with due dates d_j and release times r_j, we aim at partitioning the products into bundles and scheduling the launches of these product bundles to optimize some production function. The resulting problem, which we call the *m-vector bin packing and sequencing problem (m-BiPacS)*, can be described with a following integer program:

$$\text{min.} \quad F(\mathbf{w},\mathbf{x}) = \sum_{j \in J} \sum_{b=1}^{\bar{B}} w_{jb} \cdot x_{jb} \tag{1}$$

$$\text{s.t.} \quad \sum_{b \in L_j} x_{jb} = 1 \qquad \forall j \in J \tag{2}$$

$$\sum_{j \in J} v_{ji} \cdot x_{jb} \le c \qquad \forall i \in \{1,\dots,m\},\ \forall b \in \{1,\dots,\bar{B}\} \tag{3}$$

$$x_{jb} \in \{0,1\} \qquad \forall j \in J, \forall b \in \{1,\dots,\bar{B}\} \tag{4}$$

The binary decision variables x_{jb} equal to 1 if job j is assigned to bundle b and to 0 otherwise. The product bundles are numbered in the sequence of their launch. The upper bound on the number of the required launches is a sufficiently large number $\bar{B}$. Parameter c denotes the cycle time and parameters v_{ji} describe processing times of job j at machine i. Parameters w_{jb} represent some general cost of assigning job j to bundle b. For instance, w_{jb} may depend on the due date of the job and penalize its too late finishing. We assume the cost to be monotonic and not decrease with a later launch: $w_{jb} \le w_{jb'}$ for $b < b'$. Set L_j describes the set of bundles, whose launch is scheduled not earlier than the release time r_j of product j. The objective function is to minimize the total cost (1) while assigning each product to exactly one product bundle (2). Constraints (3) guarantee that the total processing time of the products in any bundle does not exceed the available cycle time at any of the stations $i \in \{1,\dots,m\}$.

The m-BiPacS is a generalization of the classical *vector bin packing problem (VBP)* [cf. 1, 2]. However, unlike the VBP, the sequence of bins (bundles) is relevant in the m-BiPacS as it impacts the objective function value. The m-BiPacS is also distinct from the related *batching problem (BP)* [cf. 3, 4, 5]. The BP is to partition jobs into batches and schedule batches with respect to some scheduling criterium. However, the processing times of batches in the BP are usually computed as the maximum processing time of the jobs in the batch, and the number of jobs in a batch is usually given.

2 Iterative variable neighborhood heuristic (IVNH)

The m-BiPacS is NP-hard in the strong sense. Our cooperation partners are interested in a fast solution method with the running time of a few minutes. Since IBM ILOG CPLEX 12.7.0 cannot solve realistic instances of the m-BiPacS with 100 products and more in a reasonable time, we design an *iterative variable neighborhood metaheuristic (IVNH).* The IVNH exploits the problem structure. For instance, it uses mathematical optimization to handle the assignment subproblems of the m-BiPacS that arise during the solution procedure. Moreover, because of creatively formulated neighborhoods, the IVNH examines only feasible solutions with *maximally packed* bundles, thus, most probably reducing the enumeration effort to find an optimal or a nearly optimal solution. Overall, the IVNH strikes a balance between search intensification – by iteratively performing local search with respect to three different neighborhoods – and search diversification – by jumping to a new promising area of the solution space as soon as the local optimum in all the three neighborhoods is reached.

Our extensive computational experiments confirm effectiveness of the designed metaheuristic procedure IVNH. In the performed tests, we stopped the IVNH if the incumbent solution did not improve in the last 20 iterations or the time limit of 10 minutes was reached. In case of instances with 30 products, the IVNH found optimal solutions in over 95% of instances within just a fraction of a second. The IVNH outperformed standard solver IBM ILOG Cplex for data sets with large instances of 100, 150 and 200 products, although it required just 88 seconds on average whereas the run time of IBM ILOG Cplex was limited by 10 minutes. For the total 240 test instances of 12 different parameter settings, IVNH clearly outperformed Cplex in 9 settings and found solutions of same or comparable quality in the remaining three settings. For example, in some relevant for practice data sets, the IVNH found better solutions than standard solver IBM ILOG Cplex in all the cases reducing the objective function value by more than 50% on average.

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