Cluster and Learn: Cluster-Specific Heuristics for Graph Coloring

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Abstract Graph coloring problems are NP-complete, so techniques based on *heuristics* have to be used for large-sized problems. In this paper, to solve *graph coloring problems with user requirements*, we propose a new Constraint Satisfaction Problems (CSP) based variable and value ordering heuristics learning method which is called *Cluster-Specific Heuristics for Graph Coloring* (Cluster and Learn). We explain how Cluster and Learn can be used to solve *graph coloring problems*. According to our experimental results, we show that Cluster and Learn outperforms the compared variable and value ordering heuristics.

Keywords Graph Coloring; Constraint Satisfaction Problems; Heuristics

1 Introduction

Graph coloring problems can be defined as constraint satisfaction problems (*CSPs*) [3,12], in which there are a set of variables, domains (a set of possible values of each variable), and a set of constraints. Various search techniques can be applied in order to improve the performance of CSP solvers. Variable and value ordering heuristics are common intelligent search techniques which are used by CSP solvers for solving many kinds of problems such as *configuration*, *planning and scheduling*, and *integrated circuit design* [2,7–11].

In this paper, we propose a new variable and value ordering heuristics learning approach Cluster and Learn to increase the runtime performance of solving *graph coloring problems with user requirements*. In Section 2, we show how our approach can be applied to solve graph coloring problems. The results of our evaluation are presented in Section 3. In Section 4, we conclude the paper.

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2 Cluster and Learn

Our proposed method Cluster and Learn provides runtime efficiency in solution searching for *CSPs with user requirements.* We demonstrate our approach on a *map coloring problem* [6,4] where adjacencies among 4 countries (**C1**, **C2**, **C3**, and **C4**) are **C1-C2**, **C2-C3**, **C3-C4**, and **C4-C1**.

Clustering. Cluster and Learn clusters historical user requirements (*REQs*, see Table 1) by applying *k-means clustering* [1]. Distances between two user requirements are calculated based on *euclidean distance* [13]. Since we use colors (instead of numeric values) in this example, we calculate the distance between two different colors as 1, and the distance between a color and undefined color country as 2.

	REQ_1	REQ_2	REQ_3	REQ_4	REQ_5	REQ_6
C1	blue	red	blue	red	red	red
C2	-	blue	-	blue	-	green
C3	-	-	-	-	-	-
C4	green	-	red	-	blue	-

Table 1: Six historical user requirements. "-" represents the undefined colors.

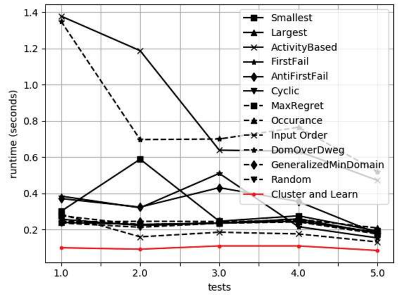
In this example, we set the number of clusters (k) to 2 (to keep the example small and understandable). After applying k-means clustering, we obtain two clusters as Cluster-1: $\{REQ_1, REQ_3, REQ_5\}$ and Cluster-2: $\{REQ_2, REQ_4, REQ_6\}$.

Learning. After clustering historical users requirements (*REQs*), Cluster and Learn run supervised learning based on a genetic algorithm [5] to minimize the runtime and finds variable and value ordering heuristics for each cluster as shown in Table 2.

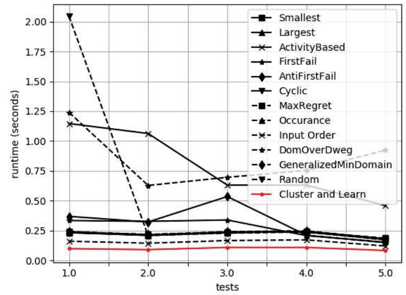
Cluster-1	Cluster-2
Variable Ordering Heuristic: **C2**, **C4**, **C1**, **C3** Value Ordering Heuristics: **C1** : $\{blue, red, green\}$, **C2** : $\{red, blue, green\}$, **C3** : $\{blue, green, red\}$, **C4** : $\{red, blue, green\}$	Variable Ordering Heuristic: **C2**, **C1**, **C4**, **C3** Value Ordering Heuristics: **C1** : $\{red, blue, green\}$, **C2** : $\{red, blue, green\}$, **C3** : $\{blue, green, red\}$, **C4** : $\{blue, green, red\}$

Table 2: Cluster-specific variable and value ordering heuristics.

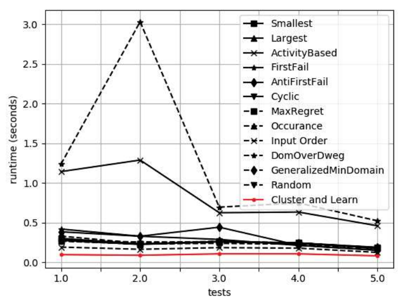
Solving. Finally, to solve $CSP_{map\_newUser}$, the CSP solver's search is guided using the variable and value ordering heuristics of the closest cluster (Cluster-1) to $REQ_{map\_newUser}$.



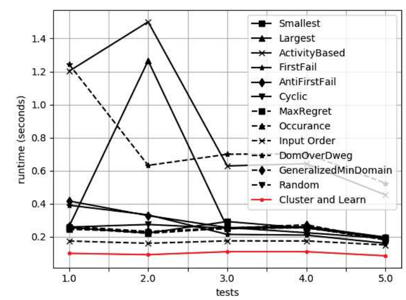
(a) Int-Domain-Min



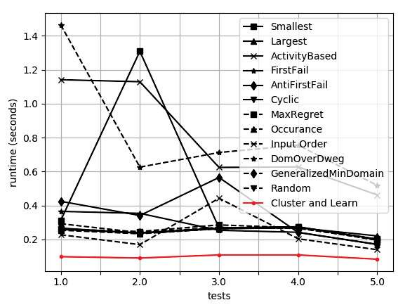
(b) Int-Domain-Max



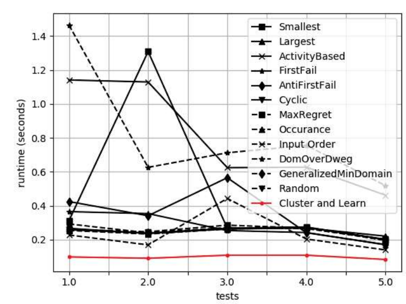
(c) Int-Domain-Median



(d) Int-Domain-Middle



(e) Int-Domain-Random



(f) Int-Domain-Random-Bound

Fig. 1: Runtime (in seconds) of the Choco solver to find first solutions. There are six charts for each value ordering heuristics of Choco Solver. In each chart, each value ordering heuristic is combined with other 13 variable ordering heuristics of Choco Solver to solve 50 different $CSP_{\text{map_test}}$s. Cluster and Learn is shown (in red lines) in all charts as a reference line (not combined with the value ordering heuristics). It is observed that among all the variable and value ordering heuristics combinations, Cluster and Learn is the fastest approach to solve $CSP_{\text{map_test}}$s.

3 Experimental Results

We have tested our proposed method Cluster and Learn on a map-coloring problem ($CSP_{\text{map\_test}}$) with 100 countries (located as 10x10 in the shape of a checkers board with 100 adjacencies). We have generated 150 different set of user requirements (each set with five random constraints) for our experiments. 100 of them are used for learning purposes and the rest 50 are used for testing the learned heuristics. As observed in Figure 1, the best performance is obtained by our approach compared to other variable and value ordering heuristics combinations.

4 Conclusions and Future Work

In this paper, to solve *graph coloring problems with user requirements*, we have proposed a variable and value ordering heuristics learning approach Cluster and Learn. According to our experiments, we observed that Cluster and Learn is outperforming all compared variable and value ordering heuristics combinations in terms of runtime performance of CSP solvers.

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