Local Search and Constraint Programming for the Post Enrolment-based Course Timetabling Problem[*]

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Abstract. We present a study of the university post-enrolment timetabling problem, proposed as Track 2 of the 2007 International Timetabling Competition. We approach the problem using several techniques, particularly local search, constraint programming techniques and hybrids of these in the form of a large neighbourhood search scheme. Our local search approach won the competition. Our best constraint programming approach uses an original problem decomposition. Incorporating this into a large neighbourhood search scheme seems promising.

1 Introduction

Timetabling problems have a wide range of applications in education, sport, manpower planning, and logistics. A diverse variety of university timetabling problems exist, but three main categories have been identified [5, 9, 26]: school, examination and course timetabling. The Post Enrolment University Course Timetabling Problem [17] occurs in an educational context whereby a set of events (lectures) have to be scheduled in timeslots and located in appropriate rooms. The problem tackled in this paper was proposed as part of the 2007 International Timetabling Competition organised by PATAT (Track 2)[1]. The problem was also used in the 2003 competition without two specific hard constraints introduced in 2007, which are discussed in Section 2. These new constraints were introduced in the 2007 competition in order to make the search for feasible timetables difficult. In 2003 finding feasible timetables was relatively easy and all algorithms, therefore, focused on optimising the soft constraints. According to the organisers [17], the two constraints have been added to move the competition closer to real-world timetabling where finding feasible timetables can be a very challenging task.

This context seemed a good opportunity to investigate Constraint Programming (CP) techniques, and compare them with the strong local search baseline developed during the 2003 challenge. Our *main contribution* in this paper is a comprehensive study of the problem using a wide range of techniques highlighting both pitfalls and positive results. Our main technical novelty lies in the analysis of complete approaches with

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[1] http://www.cs.qub.ac.uk/itc2007/

original CP models and lower bounds for the costs associated to the soft constraints, including algorithms to maintain them. We also present an original local search approach that can deal with the hardness of feasibility; this was ranked first out of thirteen teams in Track 2 of the 2007 International Timetabling Competition. Finally, a promising large neighbourhood search (LNS) scheme [27] is proposed, which contrasts with all previous published local search work on this problem [1, 6, 10, 15, 24].

2 Problem Description

The post enrolment-based course timetabling problem consists of a set of n events, E, to be scheduled in 45 timeslots $\{1, \ldots, 45\}$ (5 days of 9 hours each) using a set of m rooms, R. Each room is characterised by its seating capacity, which we will refer to as its size, and a set of features defining the set of services available in each room. Each event needs a room whose size is larger than the number of students attending the event, and it must be placed in a room with the required features. Additionally, a set of precedence requirements state that some events must occur before others. We are also given a set S of students and the set of events each must attend. Each event must be assigned to a room in a timeslot while obeying a set of constraints. The constraints of the problem are partitioned into two sets: the *hard* constraints define the requirements of a feasible timetable, while the *soft* constraints define an optimal timetable. The hard constraints are the following:

1. No student can attend more than one event at the same time.
2. In each case the room has to be big enough for all the attending students and satisfy all of the features required by the event.
3. Only one event is put into each room in each timeslot.
4. An event can only be assigned to its pre-defined "available" timeslots.
5. When specified, events have to occur in the correct order in the week.

Because feasibility can be very difficult to achieve, the organisers of the competition have introduced the notion of "distance to feasibility" to be able to discriminate entries that do not find any feasible solution. We will ignore this point in our study and consider all infeasible solutions as mere failures[2]. The quality of a feasible timetable is evaluated in terms of the soft constraints. A feasible solution is penalised equally if a student:

1. attends an event in the last timeslot of the day ($\{9, 18, 27, 36, 45\}$);
2. attends more than two events in a row on a given day; one penalty is counted for each event attended consecutively after the first two;
3. attends exactly one event during a day.

The problem defined by the hard constraints only can be seen as a constrained list-colouring in a graph where a node is an event and an edge is added between two events that must go to different timeslots. This graph is primarily made of many large overlapping cliques, referred to as *student cliques*, defined by the set of events chosen by each student. It can also be worthwhile to notice that two events that share a unique

[2] This notion is indeed relevant for the competition but not for the problem itself.

Table 1. Some statistics about the colouring graph structure in the first eight instances.

inst	student cliques			room cliques				final cliques				density	
	min	max	avg	number	min	max	avg	number	min	max	avg	basic	full
1	18	25	21.02	6	12	32	17.17	506	12	32	22.07	0.33	0.34
2	19	24	21.03	5	8	32	18.80	505	8	32	21.92	0.37	0.37
3	10	15	13.38	13	1	7	3.85	906	7	28	19.55	0.47	0.48
4	10	15	13.40	10	1	10	3.90	925	4	33	21.75	0.52	0.52
5	19	23	20.92	14	2	21	9.07	314	5	25	20.66	0.30	0.31
6	18	24	20.73	17	4	17	11.12	317	7	26	20.62	0.29	0.30
7	10	15	13.47	19	3	18	8.26	498	5	29	18.57	0.52	0.53
8	11	15	13.83	19	2	13	7.26	503	7	25	17.65	0.51	0.52

possible room, due to their size and features, have to be assigned to different timeslots. The cliques relying on those edges are referred to as *room cliques*. At last, precedences also imply differences and can be added in the graph. Table 1 gives some details about the size and number of cliques found in the colouring graph because both our LS and CP approaches will try to take advantage of them. It also shows the density of the *basic graph*, i.e. the original graph of student choices including the precedence edges, and the *full* graph, i.e. the same graph augmented with rooms.

The *final cliques* of Table 1 are obtained by the following process: the neighbourhood of a clique c, *i.e.* the set of nodes connected to all the nodes of c (but not necessarily with each other) can intersect another clique, and the corresponding intersection can, therefore, be used to extend c. The final cliques are obtained by applying such a process iteratively from the student/room cliques until a fixed point is reached. The density of the full graph is not much bigger than for the basic graph, but the added edges can significantly improve the maximum and average size of the cliques.

3 A Local Search Approach

Our local search baseline is strongly based on the work achieved during the 2003 competition and the improved results published later in [6, 15, 24]. There are, however, some differences to consider due to the increased difficulty of finding feasible timetables in the 2007 competition instances. Similar to most approaches of 2003, our local search is performed in two steps: we first try to identify a feasible solution, and then try to reduce the cost of violating the soft constraints. The originality of our local search lies mainly in finding feasible timetables. We describe both steps in more detail below.

3.1 Finding Feasible Solutions

The search for feasible solutions is performed by considering a unit cost per hard constraint violation: an infeasible timeslot or room for an event, two events sharing a student in the same timeslot, two events violating a precedence between them.

Representation of the solution. The *position* of an event is defined by a given timeslot *and* room. The solution is represented by the position of each event as opposed to the solution representation described in [24], which ignores the rooms and maintains the room violations by solving a matching problem per timeslot. Knowing if a set of events can fit in a given timeslot with respect to room availability and capacity is a bipartite matching problem (events to rooms). For efficiency reasons, the lists of events per timeslot as well as the list of all free positions in the timetable (positions where no event is currently assigned) are added to the representation.

Neighbourhood. The neighbourhood can be seen as a composite neighbourhood structure [1, 10] defined in terms of the following moves:

1. TrE: translates an event to a free position of the timetable.
2. SwE: swaps two events by interchanging their position in the timetable.
3. SwT: swaps two timeslots t_i and t_j, *i.e.* translates all events currently placed in t_i to t_j and all events in t_j to t_i.
4. Ma (Matching): reassigns the events within a given timeslot to minimise the number of room conflicts; to allow violations, a maximum matching is solved.
5. $TrE{+}Ma$: translates an event to a timeslot and evaluates if this does not violate the room constraints by checking the corresponding matching problem; if the matching is infeasible, the move is rejected.
6. Hu (Hungarian): picks a set of events $\{e_1, \ldots, e_k\}$ assigned in different timeslots ($k \leq 45$) that do not have precedences defined between them, and reassigns them optimally by solving an assignment problem with the Hungarian algorithm [16]. The violation of the hard constraints for placing each event in each timeslot is known as it does not depend on the other removed events, since they do not share precedences and only a single event is removed per timeslot. We solve $45 \times k$ maximum matching problems to evaluate the cost due to the room capacities of placing each event in each timeslot. As the number of such moves is exponential, the size of the neighbourhood is restricted to K sets ($K = 20$ in practice), including conflicting events (involved in hard constraints violations) and completed randomly.

TrE and SwT are always considered in the neighbourhood. The remaining moves are ranked in terms of their time complexities and included in the neighbourhood at a given iteration depending on a probability related to their complexity. More specifically, the probabilities are set to $p(Hu) = p(SwT) = \frac{5}{10^4}$, $p(Ma) = \frac{1}{10^3}$, $p(TrE + Ma) = \frac{1}{10^2}$. Thus, time consuming moves are performed less frequently than faster ones. The set of moves considered at each iteration, therefore, varies and the order of exploration amongst them is chosen randomly. However, for each move the exploration is performed deterministically from the last point where it was left (similar to [15]).

Search. Improving and sideways moves, which keep the current violation cost constant, are always accepted and no emphasis is put on conflicting events, except by move Hu. We believe that moves TrE and SwE are very important in our approach. They can be performed very quickly and, therefore, provide a diversification mechanism as the search is not guided by conflicting events. This also explains why we choose a solution representation that includes the room information explicitly, since this is mandatory for TrE and SwE. A simple tabu list of size $k = 10$ prevents cycling by forbidding an event being considered in a timeslot it was assigned in the last k iterations; this is similar to [6] and classic in graph colouring [11]. Finally, a pure random configuration is used to start as we found no significant benefits to starting from a greedy solution.

Intensification. As mentioned previously, the problem defined by the hard constraints can be seen as a constrained list-colouring problem in which the graph is made of many overlapping large cliques (see Table 1). The intensification step tries to take advantage of the presence of such large cliques by iteratively applying move Hu on each clique containing at least one conflicting event. All events of the clique have to be in different timeslots and define an assignment problem in the current timetable. This intensification is applied every 50000 non-improving iterations. All cliques containing a conflicting event are considered, and simplified to ignore any precedences amongst the events inside the clique. This step is applied on all the "final cliques" of Table 1.

Results. Table 2 compares a local search LS1 with a neighbourhood based only on the moves $\{TrE, SwE, Ma, TrE+Ma\}$ with the full scheme, LS2, described previously (involving a richer and randomised neighbourhood as well as the intensification step). All instances except instances 1,2,9 and 10 seem quite easy from the point of view of feasibility, and the efficiency of our improvements can be mainly seen on instances 2,9 and 10, which are much more challenging. The diversification given by the randomised neighbourhood (by favouring TrE and SwE), and the intensification given by Hu and its systematic use on the cliques, is beneficial for the hard instances. Table 2 shows the percentage of feasible solutions found over 100 runs with different seeds within the time limit[3] and the average time required by LS1 and LS2, which is computed only on runs that have found a solution. Instance 10 is the only instance that remains really "open" for feasibility as all others are solved more than 94% of the time.

Table 2. Percentage of solutions found, with average time, using a simple Local search (LS1) and our improved scheme (LS2).

	Instances	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
LS1	% solved	100	70	100	100	100	100	100	100	92	17	100	100	100	100	100	100
	avg time (s)	35.9	240.7	1.1	0.9	5.1	5.4	5.3	2.6	169.5	385	0.9	1.5	10	7.7	1	0.7
LS2	% solved	100	94	100	100	100	100	100	100	95	33	100	100	100	100	100	100
	avg time (s)	30.4	127.9	2	1.9	5.9	7.8	8	6.6	116.3	355	0.8	1.3	6.8	7.5	1.3	1.1

3.2 Finding Good Solutions

Once a feasible solution has been found, another local search optimises its soft cost.

Representation of the Solution. We extend the previous representation by adding the student view. The timetable of each student (needed for cost 2) is kept as a three dimensional matrix of size $|S| \times 5 \times 9$ where each entry is equal to the event attended by the student at the corresponding day and timeslot (if there is one, and set to -1 otherwise). Moreover, the number of events attended by each student, each day, is stored for cost 3.

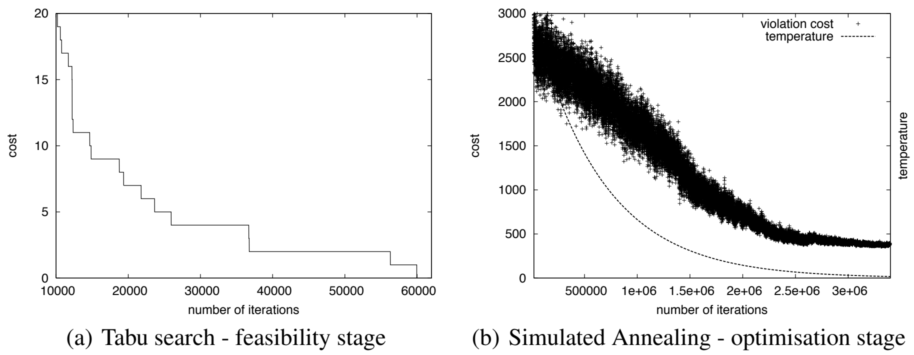
Neighbourhood. The only move used in this phase is $TrE+Ma$. Moreover, the moves considered are only those preserving feasibility. We note that this is a severe disadvantage for the search due to the tightness of the hard constraints. The main motivation for the LNS approach of Section 6 is to compensate for this disadvantage.

[3] These experiments were run on a MacBook (2 GHz Intel Core Duo, 2 GB 667 Mhz DDR2) with a time limit of 420s given by the benchmarking system of the competition.

Search. The tabu search appears inefficient for the soft cost and better results are obtained using simulated annealing (SA) [14]. This seems to match the experience of [6, 15] and the study made in [24]. Improving and sideways moves are always performed and degrading ones are accepted with a probability depending on their cost variation $\Delta : P_{acceptance}(\Delta, \tau) = e^{-\frac{\Delta}{\tau}}$ where the parameter τ, the temperature, controls the acceptance probability and is decreased over time. The temperature is *cooled* at each step using a standard geometric cooling $\tau_{n+1} = 0.95 \times \tau_n$. Two parameters are needed to define the cooling: the initial temperature τ_0, and the length of a temperature step, L, *i.e.* the number of iterations performed at each temperature level. As the time demand varies a lot from one instance to the other, we try to predict "the speed" of our soft solver during an initialisation phase by running the SA at a temperature of 1 for 20000 iterations and set τ_0 and L in the following way. Firstly, τ_0 is set to the average of the cost variation observed during the initialisation; then, based on the time needed to perform the initialisation, we get an estimation of the number of iterations that will be performed in the remaining time, I. By setting a final temperature to $\tau_f = 0.2$, we also know the number of temperature step, $nbSteps$, needed to go from τ_0 to τ_f and therefore L is set to $L = \frac{I}{nbSteps}$. A reheating is performed if the neighbourhood is scanned without accepting any moves. This can happen if the number of feasible moves is limited and the SA is more likely to reject all choices as the temperature decreases.

3.3 A Synthesis of the Local Search Approach

We conclude the presentation of the local search approach by showing the behaviour of the search at the two stages, *i.e.* feasibility and optimisation on the plots of Figure 1.



(a) Tabu search - feasibility stage (b) Simulated Annealing - optimisation stage

Fig. 1. Evolution of the violation cost per iteration for the two stages of the local search approach.

Both plots show the evolution of the costs at each iteration. The cooling is also indicated for the SA. The search for feasibility proceeds by moving over a large plateau of configurations of equivalent violation cost, *i.e.* the cost is never degraded in practice. Sideways moves appear to be very frequent for feasibility. Therefore, the search can stay for a long time on the same plateau as it does not focus on conflicting events and accept any sideways step; this is why favouring moves TrE and SwE brings diversification over the plateau. Therefore, the role of the intensification step is important.

Sideways moves are less likely to occur at the optimisation stage and one can see the effect of the cooling by observing that the cost variation is decreasing while the best known cost is converging toward its final value. The choice of the different metaheuristics for feasibility and optimisation, with their resulting behaviours, is also motivated by the fact that, in the first case, we try to get a feasible solution as soon as possible whereas in the second case we aim for the best possible solution within a given time-limit.

4 Constraint Programming Models for Feasibility

This timetabling problem was tackled by a number of local search techniques [1, 6, 10, 15, 24]. We are not aware, however, of any complete approach. We considered several CP models, none of which were able to match the efficiency of local search. However, as we shall see in Section 6, the CP approach can still be valuable to provide complex neighbourhoods within the SA algorithm. We present here the most promising CP model as well as two less successful ones and give some insights into their inefficiency.

4.1 Basic Model

For an event i we introduce two variables $eventTime_i \in \{1, \ldots, 45\}$ and $eventRoom_i \in \{1, \ldots, m\}$, for the timeslot and room associated to event i, respectively. Let R_i be the set of rooms that can accommodate event i, T_i be the set of timeslots available for event i, $student(i)$ be the set of students attending event i and, finally, let $prec$ be the set of the pairs of ordered events. We define the first model as follows:

Model 1

$$\begin{array}{lr}
\forall i,j \leq n \text{ s.t. } student(i) \cap student(j) \neq \emptyset & eventTime_i \neq eventTime_j \text{ (1)} \\
\forall i \leq n & eventRoom_i \in R_i \text{ (2)} \\
\forall i,j \leq n \quad (eventTime_i \neq eventTime_j) \vee (eventRoom_i \neq eventRoom_j) & \text{(3)} \\
\forall i \leq n & eventTime_i \in T_i \text{ (4)} \\
\forall (i,j) \in prec & eventTime_i < eventTime_j \text{ (5)}
\end{array}$$

In this viewpoint, the constraints (1), (4) and (5) correspond to a *list colouring* problem with precedences on the variables $eventTime$. Constraints (2) and (3) enforce that events be allocated to suitable rooms, and that within a given timeslot, every event be put into a different room, respectively. They correspond to a set of *matching* problems, one for each timeslot, conditioned by the result of the above colouring problem. An important observation is that these two aspects are relatively disconnected. Indeed, as long as an event is not committed to a given timeslot, we do not know in which matching it will participate because of the disjunctions (3). If an early decision on the colouring part prevents a consistent room allocation, it will not be discovered until very late in the search, leading to a *trashing* behaviour where large unsatisfiable subtrees are explored again and again. We explored two ways of resolving this issue. First, we modelled the relation between the room allocation (matching) and timeslot allocation (colouring) using a global constraint [2] to achieve stronger inference between these two aspects and detect mistakes earlier. We describe this model in Section 4.2. The second solution was to separate the solving of the colouring and the matchings, so that we explore more diverse colourings, and hopefully avoid trashing. We describe this model in Section 4.3.

4.2 Matching Constraint

Knowing if a set of events can fit in a given timeslot with respect to room availability and capacity is a bipartite matching problem (events to rooms). The objective is to remove the $eventRoom$ variables from the search space. In other words, we will make sure that constraint propagation alone ensures that an assignment of all events can be extended to a matching for each timeslot. As a result, we solve a colouring problem where we only assign events to timeslots subject to timeslot availability, precedences and such that the remaining matching sub-problems are backtrack-free.

The room allocation sub-problem can be represented in a bipartite graph $G = (V_1, V_2, E)$ where $V_1 = \{1, \dots, n\}$ is the set of events, and $V_2 = \{\langle 1, 1\rangle, \dots, \langle 45, m\rangle\}$ is the set of all pairs $\langle$timeslot, room$\rangle$. An edge $(i, \langle j, k\rangle)$ is present iff event i can be assigned to timeslot j in room k. A maximal matching of G thus represents an assignment of events to rooms satisfying constraints (2) and (3). We introduce n variables to link this matching with the colouring. $event_i \in \{\langle 1, 1\rangle, \dots, \langle 45, m\rangle\}$ denotes the timeslot and room, represented by a pair, to which event i is assigned. An $\text{ALLDIFF}(\{event_1, \dots, event_n\})$ [22] makes sure that the graph G admits a matching of cardinality n. Notice that during search, arc-consistency is achieved for all matching problems at once, giving stronger inference than considering matchings independently. Notice that we could post other constraints directly on variables $events$. However, this can be done more easily on the $eventTime$ variables. They also provide a naturally good branching scheme, since rooms have been factored out of the search space. We thus define a second model, where we substitute the variable $eventRoom_i$ with $event_i$ and channel it to $eventTime_i$ using a simple binary constraint.

Model 2

$\forall i, j \leq n \; s.t \; student(i) \cap student(j) \neq \emptyset$	$eventTime_i \neq eventTime_j$ *(1)*
$\forall i \leq n$	$eventTime_i \in T_i$ *(4)*
$\forall (i, j) \in prec$	$eventTime_i < eventTime_j$ *(5)*
$\forall i \leq n$	$event_i \in \langle R_i \times T_i\rangle$ *(6)*
$\forall i \leq n$	$eventTime_i = event_i[0]$ *(7)*
	$\text{ALLDIFF}(\{event_1, \dots, event_n\})$ *(8)*

Constraint (7) channels the variables $eventTime$ to $event$ by projecting on the first element of the pair. Notice that since arc consistency is achieved in polynomial time on the ALLDIFF constraint, an assignment of $eventTime$ satisfying Model 2 can always be extended to $event$ in a backtrack-free manner.

4.3 Alternate Colourings and Matchings

Constraint (8) of Model 2 is very costly to maintain. Therefore, we consider a decomposition similar to a logic-based Benders decomposition scheme [12]. We delay the resolution of the matchings once a colouring has been found. If the matching is infeasible, we seek another solution for the colouring sub-problem, and iterate in this way until a full solution is found. Clearly, solving the colouring part alone allows for a far more optimised and sleeker model, however, reaching a fixed point might not be easy. We first describe the lighter model restricted to the colouring and precedence constraints, and where the room allocation constraints are relaxed to a simpler cardinality constraint. Then, we show how Benders cuts can be inferred when failing to solve a matching in order to tighten the colouring sub-problem.

The $eventRoom$ variables are ignored as in the previous model and a single global cardinality constraint (GCC) [23] is added to ensure that every timeslot is used at most m times. This constraint eliminates trivially infeasible matchings where the number of events assigned to a timeslot is greater than the number of rooms.

Model 3

$\forall i, j \leq n \; s.t. \; student(i) \cap student(j) \neq \emptyset$	$eventTime_i \neq eventTime_j$ *(1)*
$\forall i \leq n$	$eventTime_i \in T_i$ *(4)*
$\forall (i,j) \in prec$	$eventTime_i < eventTime_j$ *(5)*
$\forall i \leq n$	$\text{GCC}(\{eventTime_i \mid i \leq n\}, [[0..r], \ldots, [0..r]])$ *(9)*

A solution of this model is not guaranteed to be a feasible solution of the original problem. Indeed, a matching problem can be inconsistent once the colouring is fixed. We, thus, iteratively solve the colouring part until we find a feasible room allocation, as depicted in Algorithm 1. If a matching problem fails, a minimal conflict corresponds to a set of events that cannot be assigned together in any timeslot, whilst forming an independent sub-graph of the colouring graph. We use an algorithm for finding minimal conflicts [8] to extract such a set of events (line 3). In order to rule out this conflicting assignment in future resolutions of the colouring sub-problem, we post a NOTALLEQUAL constraint to the model (line 4). The constraint NOTALLEQUAL$(x_1, \ldots x_k)$ ensures that there exists $i, j \in [1..k]$ such that $x_i \neq x_k$. It acts as a Benders cut and prevents the same assignment from being met again. Observe that since we extract minimal sets of conflicting events [4, 13], entire classes of assignments that would fail for the same reason are ruled out. Notice also that although this constraint is inferred from a particular timeslot, it holds for every timeslot.

Algorithm 1: `Decomposition`

```
1  repeat
2      solve Model 3;
       $matched \leftarrow$ true;
       foreach $1 \leq j \leq 45$ do
           $G \leftarrow (V_1 = \{i \mid eventTime_i = j\}, V_2 = \bigcup_{i \in V_1} R_i, E = \{(i,k) \mid i \in V_1, k \in R_i\})$;
           if cannot find a matching of $G$ then
               $matched \leftarrow$ false;
3              $cut \leftarrow$ Extract-min-conflict($G$);
4              add NOTALLEQUAL($eventTime_k | k \in cut$) to Model 3;
   until matched;
```

We explored further improvements of this model based on the analysis of the colouring graph described in Section 2. Conflicts between events are organised into large cliques, one for each student and even larger cliques can be inferred by taking room conflicts into account. This information can be used to obtain stronger filtering from the model. One possibility is to replace the constraints (1) by ALLDIFF constraints. Each of the aforementioned "final cliques" implies an ALLDIFF constraint between a set of $eventTime_i$ variables. In this manner, all the binary differences (1) are covered by at least one clique and can thus be removed. We can expect to achieve a stronger level of propagation as a result. On the other hand, ALLDIFF can be expensive to maintain. We can therefore choose to keep, amongst the final cliques, only the cliques obtained from a room clique, as a trade-off between the efficiency of binary differences and the additional reasoning brought by the cliques, as they are big and they contain additional conflicts. This leads to two variations of Model 3 that we assess empirically below.

4.4 Experimental Results

We ran Model 2, Model 3, Model 3-cliques (Model 3 including all implied ALLDIFF constraints) and Model 3-rooms (Model 3 including only the ALLDIFF constraints standing for room cliques). In Table 3, we give the number of iterations of Algorithm 1 (`Decompositon`), that is, the number of feasible colourings that were required to find a complete solution. This number is always 1 for Model 2. We also give the cumulative CPU time and number of nodes explored on solved instances. Notice that no model could solve instances 1, 2, 9, 10, 13 or 14 within the time cutoff of 420 seconds, corresponding to the 10 minutes cutoff of the competition on an Apple MacBook. Model 2 does not need to solve several colouring problems, however, the overhead due to the extra variables ($event$) and to the large ALLDIFF constraint, is too large. In fact the search tree explored by Model 2 is several orders of magnitude smaller than that explored by Model 3. We also observe that in most cases, the ALLDIFF constraints on events sharing the same unique suitable room reduces dramatically the number of iterations required to solve the problems. On the other hand, using ALLDIFF constraints for representing the colouring problem seems to be slightly detrimental. The best combination seems to be Model 3 using ALLDIFF only for rooms. We believe that the main reason for Model 3 to dominate Model 2 is that the difficult part of the problem lies primarily in the colouring for these instances. The very low number of colouring sub-problems solved when adding the implied ALLDIFF constraints provides further evidence of this. Any given colouring satisfying the implied ALLDIFF constraints is very likely to be extensible to a feasible matching. We also observed (but this is not apparent in the tables) that the extra GCC constraint used to approximate the matching part was almost unnecessary in most cases. That is, even without this constraint, the number of iterations to reach a complete solution remains relatively small. Notice, however, that this last observation does not stand for instances 1, 2, 9 and 10, which happen to be the hardest.

Next we compare three heuristics all using the best model: Model 3 (room). We used *minimum domain over future degree (dom/deg)* and *impact* [21] as benchmarks, since they are both good general purpose heuristics. The former was used successfully on list-colouring problems in the past, whilst the latter proved to be the best in our experiments. The third heuristic, *contention*, is based on computing the contention of events for a given timeslot. In scheduling, resource contention has been used as heuristic with success [25]. In our case, timeslots can be viewed as resources, of capacity m, required by events. The *contention* $C(j)$ of a time slot j is $C(j) =$

Table 3. A comparison of the various CP models we studied.

	Model 2			Model 3 (conflicts)			Model 3 (all)			Model 3 (room)		
Inst.	iter	time	nodes	iter	time	nodes	iter	time	nodes	iter	time	nodes
3	1	12.813	327	2	5.111	312	1	15.919	198	1	4.850	198
4	-	-	-	2	7.386	3789	1	18.154	351	1	4.814	351
5	-	-	-	-	-	-	4	23.428	5335	3	9.599	1977
6	-	-	-	22	28.878	26049	3	93.895	42753	2	25.619	22137
7	-	-	-	9	17.608	21410	1	17.459	2595	1	7.690	5626
8	1	119.190	2144	6	2.521	534	1	7.283	633	1	3.015	633
11	-	-	-	5	4.297	713	3	18.485	443	3	5.678	1896
12	-	-	-	8	160.732	178437	2	271.381	29291	1	75.705	78666
15	-	-	-	10	2.528	601	2	6.096	191	2	2.783	191
16	1	4.883	213	12	2.477	1143	2	6.153	261	2	2.713	261

Table 4. Comparison of search heuristics for the CP models.

	Impact			Contention			Dom/Deg		
Inst.	iter	time	nodes	iter	time	nodes	iter	time	nodes
3	1	4.850	198	1	3.455	182	1	3.183	228
4	1	4.814	351	-	-	-	-	-	-
5	3	9.599	1977	3	66.489	112413	-	-	-
6	2	25.619	22137	2	318.635	529877	-	-	-
7	1	7.690	5626	-	-	-	-	-	-
8	1	3.015	633	2	1.958	413	3	3.021	3098
11	3	5.678	1896	4	3.165	342	-	-	-
12	1	75.705	78666	-	-	-	-	-	-
15	2	2.783	191	1	6.224	6478	-	-	-
16	2	2.713	261	2	1.878	252	2	1.831	237

$\sum_{i \mid j \in D(eventTime_i)} 1/|D(eventTime_i)|$. Intuitively, this quantity describes the demand for timeslot j. It clearly induces a value ordering, since less contended for time slots are less likely to lead to a failure. Next we can compute a contention value for variables $C(eventTime_i)$, representing how constrained is a given variable and equal to $C(eventTime_i) = \sum_{j \in D(eventTime_i)} \frac{1}{C(j)}$. The event i that minimises $C(eventTime_i)$ and the timeslot j that minimises $C(j)$ are explored first.

In Table 4, we give the number iterations of `Decompositon` (Alg. 1) as well as the cumulative cpu time and number of nodes explored on solved instances. The results clearly show that *contention* dominates *dom/deg* and is itself dominated by *impact*. Notice that these two better heuristics also provide value orderings, whereas *dom/deg* does not. This is important on these benchmarks, since they have a relatively large number of solutions whilst being hard for a complete method.

5 Constraint Programming Models for Optimisation

In this section we introduce three soft global constraints to reason about the costs and especially derive lower bounds. The main difficulty we encountered is that all three costs are defined in terms of students who are numerous and, thus, not represented explicitly in our CP model. In each case we tried to get around this issue by projecting the cost on events and/or timeslots.

5.1 Last Timeslot of each Day

This soft constraint counts the number of students attending an event in the last timeslot of the day ($\{9, 18, 27, 36, 45\}$). Let us introduce, for each event i a Boolean variable b_i such that $b_i = 0$ if the event i is in a timeslot other than the last ones, and $b_i = 1$ if the event i is in one of the last timeslots. The cost can then be expressed as $cost_1 = \sum_i (b_i \times |student(i)|)$. The Boolean variables can be added to Model 3 and channelled with $eventTime_i$ or a simple dedicated global constraint can be implemented. We chose the latter option for efficiency reasons and to be able to augment it with stronger inference.

Lower Bound. Consider the bipartite graph $G = (V_1, V_2, E)$ described in Section 4.2 and captured by constraint (8) of Model 2. We recall that $V_1 = \{1, \ldots, n\}$ is the set of events and $V_2 = \{\langle 1, 1\rangle, \ldots, \langle 45, m\rangle\}$ is the set of all pairs $\langle$timeslot, room$\rangle$. The existence of a maximum matching in this graph ensures a possible allocation of each event to a pair $\langle timeslot, room\rangle$. WG extends G by adding a weight w_{ij} to each edge of E defined as follows:

$$w_{ij} = \begin{cases} 0 & \text{iff } j = \langle a, b\rangle \text{ with } a \in \{9, 18, 27, 36, 45\}; \\ |student(i)| & \text{otherwise.} \end{cases}$$

Let us denote by W the value of the maximum weighted matching in WG. Observe that W represents the maximum number of students who can fit in the 40 non-last timeslots and the rest is therefore a lower bound on the minimum number of student going in the last timeslots: $lb(cost_1) = \sum_{i \leq n} |student(i)| - W$.

Pruning. The pruning process is trivial here and removes values $\{9, 18, 27, 36, 45\}$ from the domain of an event $eventTime_i$ if $lb(cost_1) + |student(i)| > ub$ and event i has not been included in $lb(cost_1)$. This can be done in $O(n)$ time.

Computational Complexity. The maximum weighted matching corresponds to an assignment problem and can be solved in polynomial time (in $O(n^3)$ with the Hungarian method [16]). As n can reach 400 in the data sets, an incremental algorithm for the maximum weighted matching is needed and this improved bound has not yet been included in our current implementation.

Note that this bound is exact when relaxing only constraints 1 and 5 of the problem description. We have seen that the colouring sub-problem can however be tighter than the matchings so that reasoning on the colouring might improve this bound.

5.2 Consecutive Events

This soft constraint counts the number of students attending more than two events in a row on a given day. The main difficulty with this cost is the potentially large number of parameters having an impact on the cost. We present the lower bound developed for this cost and how it is maintained incrementally at a relatively low computational cost.

Lower Bound. We first consider only events committed to a timeslot, *i.e.*, instantiated variables. The cost of consecutive allocation of every possible triplet of events is pre-computed initially and stored in a large static table: $static\text{-}cost(i_1, i_2, i_3) = |student(i_1) \cap student(i_2) \cap student(i_3)|$. The lower bound, $lb(cost_2)$, is then, firstly, made of the sum of these costs implied by instantiated events i_1, i_2, i_3 to consecutive timeslots. This part of the bound is referred to as $lb_g(cost_2)$:

$$ground\text{-}cost(i_1, i_2, i_3) = \begin{cases} static\text{-}cost(i_1, i_2, i_3) & \text{if } eventTime_{i_1/i_2/i_3} \text{ are} \\ & \text{assigned and consecutive;} \\ 0 & \text{otherwise.} \end{cases}$$

$$lb_g(cost_2) = \sum_{i_1 < i_2 < i_3} ground\text{-}cost(i_1, i_2, i_3).$$

Then, for each unassigned event, a lower bound on the cost involved by its insertion in the current timetable is maintained. For a timeslot j, let $pairs(j)$ be the set of pairs of events assigned respectively to $j-2$ and $j-1$, or $j-1$ and $j+1$, or $j+1$ and $j+2$. The cost of assigning event i to timeslot j is the sum of all triplets formed by i and any existing pair p in $pairs(j)$. Then we define $pending\text{-}cost(i, j)$ as: $\sum_{p \in pairs(j)} static\text{-}cost(p \cup \{i\})$. The lower bound $lb(i)$ associated with allocating event i to one of its possible timeslots is equal to the minimum pending cost over all values $lb(i) = min_{j \in D(eventTime_i)} pending\text{-}cost(i, j)$. We use the following lower bound during search:

$$lb(cost_2) = lb_g(cost_2) + \sum_{|D(eventTime_i)|>1} lb(i).$$

Pruning. We prune timeslot j for event i iff $lb(cost_2) + pending\text{-}cost(i, j) - lb(i)$ is greater than the current upper bound of the variable associated to this cost.

Computational Complexity. The base lower bound $lb_g(cost_2)$ is maintained incrementally during search. It is updated only when a variable $eventTime$ becomes assigned to some timeslot. In this case we increase the cost by the the value of $static\text{-}cost$ of the newly formed triplets of events. There are at most $35m^3$ triplets in total, the amortised computational cost of maintaining this lower bound along one branch of the search tree is thus $O(m^3)$. The pre-computation of the static-cost is here a key for efficiency. Computing $pending\text{-}cost(i, j)$ can be done in $O(m^2)$ time since there are three sets of at most m^2 pairs to consider for each timeslot of each event. Since there are at most 45 possible timeslots for a given event, one can compute $lb(i)$ for all events in $O(nm^2)$. In practice, we update the values of $lb(i)$ only when event i loses some values, or when another variable get assigned to some timeslot j and $D(eventTime_i) \cap \{j-2, j-1, j+1, j+2\} \neq \emptyset$. The pruning can be done in the same time complexity since we only need to go through at most $45n$ values.

Alternative Lower Bound. For a given student s, let us introduce the following Boolean variables: for each timeslot j, $b^s_j = 1$ if the student has an event assigned to the timeslot j, $b^s_j = 0$ if he is free at that time. These variables can be easily channelled with the $eventTime_i$ variables. For a given assignment of $b^s_1, \ldots, b^s_{45}$, c^s is the corresponding cost (*i.e.* the sum of the number of triplets by day), and for a given $c \geq 0$, $opt^s(c) = max\{\sum_{j \leq 45} b^s_j \mid c^s \leq c\}$. In other words, $opt^s(c)$ is the biggest number of b^s_j variables we can set to 1 without exceeding the cost c.

A lower bound $lb'(s)$ of the cost for the student s can thus be defined as follows:

$$lb'(s) = min\{c \geq 0 \mid |events(s)| \leq opt^s(c)\}$$

i.e. the minimal cost at which we can place all the events of the student s. The alternative lower bound for the cost is therefore $lb'(cost_2) = \sum_{s \in S} lb'(s)$.

Proposition 1. *Let c be a value* ≥ 0*, in a given state of the* b^s_i *variables, determining* $opt^s(c)$ *is polynomial.*

Proof. Consider each timeslot of one day from the first to the last and the following greedy procedure. If an event can be added in the current timeslot without creating more triplets than c, add it, else, let this timeslot empty. Repeat this for each day. The number of events finally added is optimal. Indeed, suppose we are at the timeslot k in some day, and let $a(k-1) = \sum_{j \leq k-1} b^s_j$, a be the total number of b^s_j set to 1 when $b^s_k = 1$ and a' when $b^s_k = 0$. Let us prove $a \geq a^{'}$.

If $b^s_k = 1$ increases the number of triplets over c, clearly we have no other choice than setting it to 0. Suppose it is not the case. If we set b^s_k to 1, then in the worst case, that is if $b^s_{k-1} = 1$, we must let $k+1$ empty and optimally fill the remaining timeslots with a_r 1's. Thus $a \geq a(k-1) + 1 + a_r$. If we set b^s_k to 0, then the remaining slots can be optimally filled with a'_r 1's, and so $a' = a(k-1) + a'_r$. We have $a'_r = a_r + 1$ or $a'_r = a_r$, so $a \geq a(k-1) + 1 + a_r \geq a(k-1) + a'_r \geq a'$. So, supposing (by induction) that $a(k-1)$ was optimal, we optimise the total number of 1's by setting b^s_k to 1 whenever that is possible. □

This is of course not an exact lower bound, as we do not take into account that one event can only go to a single timeslot, as nothing prevents us from putting the same event into two different timeslots for two different students in order to optimise the cost of each of them. We also do not take into account the domains of the events.

Proposition 2. $lb(cost_2)$ *and* $lb'(cost_2)$ *are incomparable.*

Proof. $lb(cost_2)$ *is not better than* $lb'(cost_2)$*:* Consider three events $1, 2, 3$ such that $students(1) \cap students(2) \cap students(3) = \{s\}$. Suppose that we have $cost_2 \leq 0$ and $eventTime_1, eventTime_2 \in \{1, 2\}$, $eventTime_3 \in \{3\}$. We have $lb_g(cost_2) = 0$, $lb(1) = 0$, $lb(2) = 0$, hence the overall lower bound is $lb(cost_2) = 0$. However using the alternative cost, when considering the student shared by all three events, we have $b^s_1 \in \{0, 1\}$, $b^s_2 \in \{0, 1\}, b^s_3 = \{1\}$. Supposing we only have a single day with three timeslots (we can easily repeat this basic pattern to fill the whole week for a more realistic situation), $lb'(s) = 1$, hence $lb'(cost_2) > 0$.

$lb'(cost_2)$ *is not better than* $lb(cost_2)$*:* Suppose now we have two students indexed 1 and 2, such that the first one is busy at timeslots 1,2 and 6, and the second at timeslots 1, 5 and 6. We have one more event taken by both students that can go in 3 or 4. Then $lb'(1) = lb'(2) = lb'(cost_2) = 0$, by setting for each student respectively $b^1_3 = 0$, $b^1_4 = 1$ and $b^2_3 = 1$, $b^2_4 = 0$. However this does obviously not lead to a solution, which would have been detected by $lb(cost_2)$. Indeed, $lb(3) = lb(4) = lb(cost_2) = 1$. □

5.3 Single Events

This soft constraint counts students attending a single course in any day of the week. The non-monotonic nature of this cost makes it difficult to reason about. Indeed, scheduling an event in a given day simultaneously increases the cost for students attending only this event, and decreases it for student attending another, until then unique, event. In fact, we show that even when we relax all other factors to an extremal case, computing an exact lower bound for this constraint is NP-hard.

Theorem 1. *Finding the exact lower bound for Cost 3 is NP-complete, even if all other constraints are relaxed.*

Proof. We consider the problem of finding a lower bound for $cost_3$ for a given day, with as few external constraints as possible. We only assume that a set of events may already have been assigned to this day, and that we have a finite set of extra events to choose from. We analyse the corresponding decision problem, SINGLE-EVENT:

Data: An integer k, a set R of events already assigned to a given day, and another set P that can possibly be assigned to this day.
Question: Is there a set $R \subseteq S \subseteq P$ of events such that no more than k students have a single event in that day.

We reduce SET-COVER to SINGLE-EVENT. A SET-COVER instance is composed of a set $U = \{u_1, \dots, u_n\}$, a set $S = \{S_1, \dots, S_M\} \subseteq 2^U$ of subsets of U, an integer $k \leq M$. The problem consists of deciding whether there exists a set $C \subseteq S$ such that $\cup_{S_i \in C} S_i = U$ and $|C| \leq k$.

We build R with one event E, that contains $k+1$ students $e_i^1, \dots, e_i^{k+1}$ per element u_i of U (the element-students). We build P with an event E_j for each subset $S_j \in S$. Each event E_j contains the element-students of each element in S_j, i.e. the element-students $e_i^1, \dots, e_i^{k+1}$ for each $u_i \in S_j$, plus one unique student s^j (the set-student).

Each subset $R \subseteq S \subseteq P$ of cost k, i.e. such that no more than k students attend a single event in the day, corresponds to a set cover of U of size k and vice-versa. A set cover C of size k corresponds to a set $S = \{E_j | S_j \in C\}$ of cost k. Conversely, a set S of cost k corresponds to a set cover of size k. Clearly, S corresponds to a set cover: if any element of U is not covered, then the cost of S is at least $k + 1$ (each uncovered element corresponds to $k + 1$ element-students attending only E). Now, as all the element-students attend at least two events, the cost can only result from the set-students, which is simply the number of events (other than E) in S. $\square$

Observe that solving a sequence of SINGLE-EVENT instances with decreasing values of k gives us a lower bound on this cost when all other constraints are relaxed, and *without even imposing each variable to take at least one value*. For instance if event i is in P, we can choose not to schedule it at all, whereas in effect, it will necessarily be assigned to some day of the week. This is, therefore, a much easier problem than finding the exact lower bound of Cost 3. However, even this relaxed problem is NP-complete.

Taking this fact into consideration, we only maintain this cost correctly in the computationally cheapest possible way. We consider each pair $\langle$day, student$\rangle$. As long as at least two events attended by this student can potentially happen this day, we do nothing.

Otherwise, there are two choices, either this student has no course at all in this day, or has exactly one. In the latter case we increase the cost by one. This can be efficiently done with a system akin to the *watched literals* used in SAT unit propagation [18]. For every student and every day, we randomly pick two events to "watch" for this pair. When an event cannot be assigned to some day anymore, we update its list of students watched for that day, finding a new available watcher. Notice that this is very cheap to do. For instance if this event was not watching any student for that day, it does not cost anything at all. When we do not find any replacement, we know that the given student is either attending no event in that day, or only a single one. We update the cost accordingly.

6 Large Neighbourhood Search

One weakness of the local search approach is the lack of flexibility when moving in the space of feasible solutions. The search space accessible from a given feasible solution might be very limited by the hard constraints and even disconnected. In such a case, the search can only reach the best solution connected with the initial one.

One solution would be to relax feasibility during search without any guarantees to find it again or to restart from different feasible solutions. Due to the difficulty of finding feasible solutions, we discarded these two approaches. Another alternative is to design more complex moves that affect larger parts of the current assignment. Move Hu is one example of a complex move that remains polynomial. A more general kind of move can be performed using a complete solver. This is the central idea of Large Neighbourhood Search (LNS) [27]. LNS is a local search paradigm where the neighbourhood is defined by fixing a part of an existing solution. The rest of the variables are said to be *released* and all possible extensions of the fixed part define the neighbourhood which is usually much larger than the one obtained from classical and elementary moves. Algorithm 2 presents the simple LNS scheme. An efficient systematic algorithm is needed to explore this large neighbourhood and the CP Model 3 presented earlier will be used for this.

Algorithm 2: `LNS Scheme`

```
1 find a feasible solution;
2 while optimal solution not found or time limit not reached do
3     choose a set of events to release;
4     freeze the remaining events to their current position;
5     if search for an improving solution then
6         update the upper bound;
```

Nature and Size of the Neighbourhood. The selection of variables to release is a key element of a LNS scheme. We need to decide *which* events should be released (nature of the neighbourhood), and in *what number* (size of the neighbourhood). Previous work on LNS [7, 19, 20] outlines the importance of structured neighbourhoods dedicated to the problem. We have investigated neighbourhoods that release events per timeslots (all events contained in a given set of timeslots). It is critical to choose a neighbourhood that releases *related* variables, *i.e.* variables that are likely to be able to change and exchange their values. It is indeed very important that the neighbourhood contain more feasible solutions than the one we already had before releasing the variables. A promising neighbourhood is also likely to contain feasible solutions of better cost. We, therefore, investigated a neighbourhood that releases kc conflicting and kr random timeslots (all events in the corresponding timeslots are released).

The size of the neighbourhood is difficult to set as the tradeoff between searching more versus searching more often is difficult to achieve. We choose to start from small sizes ($kc = 2$ and $kr = 2$) and to increase it when the search stagnates; in practice, after 100 non improving iterations, the minimum of kc and kr is increased by 1. The reason is that the accurate size seems to vary a lot regarding instances. Much bigger sizes are typically needed for instances 1, 2, 9 and 10 where feasibility is tight. Sideways moves are again very important for diversification and are always accepted.

LNS as an intensification mechanism for the SA. The LNS approach relies only on the CP solver as shown on Algorithm 2. Another idea is to use the LNS move at the low temperatures of the SA to help the very important and final phase of optimisation performed at the end of the cooling. In this mode, we do not accept sideways moves to speed up the solving process and look for improving solutions only. Diversification is ensured by other moves of the SA that continuously change the current assignment. The CP move is included in the neighbourhood of the SA at each iteration with a probability that increases while the temperature decreases: $p_{include\_lns}(\tau) = \frac{1}{200*\tau}$.

7 Experimental Results

Comparison of our Approaches. We summarise here the results of our study with four approaches:

- CP: The Constraint Programming approach described in Sections 4 and 5 based on Model 3 (room) using Impact-based search.
- LNS: The Large Neighbourhood Search approach relying on Model 3 (room) (the local search of Section 3.1 is used to provide an initial feasible solution).
- SA: The local search approach described in Section 3 which is based on Simulated Annealing for the optimisation stage.
- SA_LNS: the SA approach augmented with LNS as an intensification mechanism at the end of the cooling (still using Model 3 (room)).

Table 5 reports the cost found by each technique on the 16 instances. LNS, SA and SA_LNS were run on 20 different seeds and the average, min and max cost found over the 20 runs are reported. The CP approach is entirely deterministic and a single run is therefore shown. The last three columns show the percentage of improvements given by SA_LNS over SA alone. Two computers were used, CP was run on a MacBook[4] within a time limit of 420s and the others were run on an iMac[5] within a time

[4] Mac OS X 10.4.11, 2 GHz Intel Core 2 Duo, 2 GB 667 Mhz DDR2.

[5] Mac OS X 10.4.11, 2.33 GHz Intel Core 2 Duo, 3 GB 667MHz DDR2 SDRAM.

limit of 372s[6]. Firstly, the LNS scheme outperforms CP alone (even by only looking at instances where CP does find a feasible solution) while being a very simple modification of CP. Secondly, LNS is itself outperformed by the SA. We observed here that it remains stuck in local minima despite the large size of the neighbourhood. [19] outlines the same problem and suggests that the LNS scheme still needs other and more powerful diversification mechanisms. Finally, SA_LNS improves LNS but is not very convincing. The CP moves do not seem to bring much more flexibility to the SA to escape local minima in general. It allows, however, to find three new optimal solutions (instances 7, 12 and 16) and improves significantly the resolution of two instances (7 and 16). Note that all the minimum costs are improved showing that LNS does play a role in the final intensification stage even if this does not give a major improvement.

Table 5. Overall results on 20 runs reporting the average, min and max cost.

Inst	CP	LNS			SA			SA_LNS			Improvements		
		avg	min	max	avg	min	max	avg	min	max	%avg	%min	%max
1	-	2042.90	1758	2365	1012.75	597	1339	1006.50	593	1324	0.62	0.67	1.12
2	-	2255.16	2062	2561	1414.00	758	2462	1429.68	758	2446	-1.11	0.00	0.65
3	1930	778.00	494	1041	232.50	166	309	225.25	155	307	3.12	6.63	0.65
4	2097	950.70	776	1110	358.95	249	420	355.40	242	425	0.99	2.81	-1.19
5	1767	962.80	755	1133	3.50	0	10	4.15	0	11	-22.06	0.00	-10.00
6	1681	984.00	845	1136	11.90	0	76	13.45	0	80	-13.03	0.00	-5.26
7	1450	561.70	308	719	11.80	6	53	0.60	0	6	94.92	100.0	88.68
8	1111	498.15	389	644	0.00	0	0	0.00	0	0	0.00	0.00	0.00
9	-	2402.00	2168	2708	1972.77	1099	2660	1950.55	1099	2620	1.13	0.00	1.05
10	-	2572.22	2097	2940	2209.22	1515	2730	2250.75	1512	2730	-1.88	0.20	0.00
11	2388	852.65	618	1120	347.80	242	538	334.05	241	462	3.95	0.41	14.13
12	2328	1156.30	992	1368	393.50	1	606	386.45	0	602	1.79	100.00	0.66
13	-	1011.45	839	1175	113.30	0	218	113.55	0	195	-0.22	0.00	10.55
14	-	1087.00	960	1189	0.90	0	3	1.05	0	3	-16.67	0.00	0.00
15	1225	620.45	492	795	71.15	0	269	71.10	0	268	0.07	0.00	0.37
16	964	456.65	342	541	27.35	1	135	16.20	0	130	40.77	100.00	3.70

Comparisons with other Algorithms in the Competition. Five algorithms[7] were choosen for the final phase and evaluated on 24 instances (the 16 already mentioned and 8 unknown competition ones). Since all solvers were randomised, 10 runs per instance were performed giving 50 runs per instance. Each run was ranked among the 50 for each instance and the average rank across all runs and all instances was used to give a rank to each algorithm. Table 6 shows the ranking of each algorithm, with the number of times they have found the best solution among all runs for a given instance, and the number of times they have failed to find a feasible solution.

Our local search with a score of 13.9 therefore did significantly better than the algorithm of Atsuta et al. that came second with 24.43. Our approach appeared generally more robust for both finding feasible and good solutions (Chirandini et al. being the most robust on feasibility only). It also obtained the best results on many instances. It is however very interesting to notice that it was outperformed on instances we considered here as very hard on feasibility for our algorithm, e.g. instance 10. Nothegger et al. or

[6] Both time limits were established by the benchmarking program used during the competition.

[7] The other four algorithms were developped by: M. Atsuta, K. Nonobe, T. Ibaraki; M. Chiarandini, C. Fawcett, H. H Hoos; C. Nothegger, A. Mayer, A. Chwatal, G. Raidi; T.Muller.

Table 6. Ranking of the five finalists from the tests ran by the organizers on the 24 instances.

	Atsuta et al.	Cambazard et al.	Chiarandini et al.	Nothegger et al.	Muller
Average rank (out of 240 runs)	24.43	**13.9**	28.34	29.52	31.31
Number of best solutions (out of 24 instances)	11	**13**	3	11	0
Number of failures on feasibility (out of 240 runs)	43	17	**4**	54	53
Rank in the competition	2	**1**	3	4	5

Atsuki et al. could not only systematically find a feasible solution on instance 10 but also the optimal one. On the other hand, these algorithms fail to find feasible solutions on instances where our approach succeeds easily. On instance 9 they only find a feasible solution 30% of the time but when they do, it is the optimal one. This lead us to conjecture that they have been using the soft cost to guide the search of feasible solutions. On very tight instances, this strategy pays off as there are maybe few feasible solutions and it is known that an optimal solution of cost 0 always exists. On other instances it either misleads the search or just slows down the process, thus degrading the results. This shows that there is a significant room for improvement in our results.

8 Conclusion

We have presented a comprehensive study of a university timetabling problem, comparing a variety of local search and constraint programming approaches. We designed a constraint programming approach that proceeds by decomposing the list-colouring and the matching subproblems and outperforms more classical CP models. Lower bounds were introduced to tackle soft constraints, leading to the first complete algorithm for this problem. While our local search technique benefits from the experience of the 2003 competition, we have presented several improvements to deal with hard constraints; the results show more maturity than the CP technique. However, an LNS scheme integrating both our CP and LS approaches obtained the best results. The structure of the list-colouring graph made of large and overlapping cliques was shown to be important for both CP and LS techniques. Improving the propagation we can achieve from a collection of ALLDIFF constraints is very important in this context. Arc-consistency on two overlapping ALLDIFF is already known to be NP-Complete [3] but a number of pragmatic filtering rules could be designed. This is an important topic for future work.

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