Polynomial-time Personnel Scheduling with Soft Constraints

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1 Introduction

Personnel scheduling is an optimization problem in which shifts are assigned to employees, considering a set of constraints and optimization objectives varying greatly in different applications. Although many personnel scheduling problems are NP-hard [2], some cases have been shown to be solvable in polynomial time [3, 5, 6]. These special problems with typical constraints and objectives are important to gain a deeper understanding of personnel scheduling and to develop efficient algorithms [7].

On the other hand, considering the flexibility often required in practice when constructing personnel schedules, soft constraints, which may be violated at the cost of a predefined penalty, are necessary in many scheduling applications. To the best of our knowledge, most polynomial solvable models for personnel scheduling are presented without including soft constraints. In this paper, we show how to extend the minimum cost network flow model for personnel scheduling problems with soft constraints, using dummy nodes and arcs functioning as slack variables.

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Proceedings of the 12th International Conference on the Practice and Theory of Automated Timetabling (PATAT-2018), Vienna, Austria, August 28–31, 2018

2 The minimum cost network flow model

The minimum cost (min-cost) network flow model is a widely-used efficient formulation of personnel scheduling problems [3, 5, 6]. To show the effectiveness of this model, we take problem P_1 as an example: given shifts $k \in K$, days $j \in J$ and employees $i \in I$,

- **Objective**. Minimizing the sum of cost ($c_{i,j,k}$) incurred by assigning shift k on day j to employee i.
- **Assignment constraints**. Any employee i is assigned at most one shift per day.
- **Staffing constraints**. For shift k on day j, there is a minimum ($d^l_{j,k}$) and maximum ($d^u_{j,k}$) staffing demand.
- **Working time constraints**. The number of days that employee i works should equal the number (a_i) defined in the employee's contract.
- **Skill constraints**. Shift k on day j can only be assigned to the subset of employees who are qualified for this work ($i \in S_{j,k} \subseteq I$).

Figure 1 shows the min-cost network flow model for P_1 with $|I| = 3$, $|J| = 2$, $|K| = 2$, which was first proposed in [6]. There are a source node s with a supply $\sum_{i \in I} a_i$, shift nodes $<j,k>$ associated with day j and shift k, work nodes $<i,j>$ associated with employee i and day j, employee nodes $<i>$ and a sink node f with a demand $\sum_{i \in I} a_i$. The minimum cost of flows in the network equals the minimum total cost of the assignments, as the flow from the shift node $<j,k>$ to the work node $<i,j>$ with a cost $c_{i,j,k}$ corresponds to the assignment that employee i works shift k on day j. The arc from $<j,k>$ to $<i,j>$ exists if $i \in S_{j,k}$, corresponding to the skill constraints. The capacity constraints of arcs from the source node s to $<j,k>$, arcs from $<i,j>$ to $<i>$ and arcs from $<i>$ to the sink node f imply staffing constraints, assignment constraints and working time constraints respectively. Since all arc capacities and supplies/demands of nodes are integer, the minimum cost network flow model can be solved in polynomial time to get integer solutions [1].

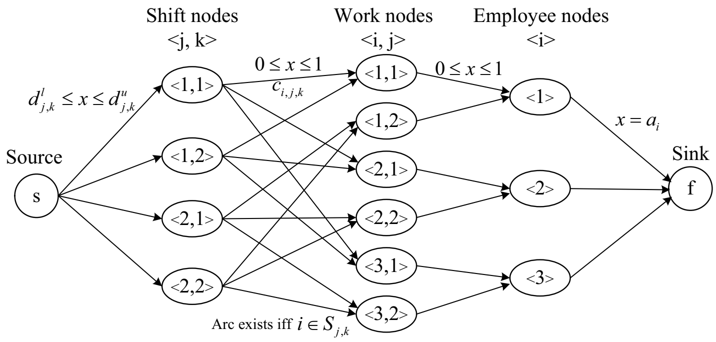

Fig. 1: The min-cost network flow model for problem P_1

3 Modeling personnel scheduling with soft constraints

The model discussed in Section 2 lacks the required flexibility provided by soft constraints. For example, an employee may be required to work overtime with additional payments for a variety of reasons. We propose an extension of the min-cost network flow model introduced in [6] to consider soft constraints without losing the polynomially solvable property of the model. The following soft constraints are considered:

- **Soft staffing constraints**. For shift k on day j, the number of assigned employees can be out of the range of the minimum ($d^l_{j,k}$) and maximum ($d^u_{j,k}$) staffing demand, with an under-staffing penalty (v^l_{jk}) and over-staffing penalty (v^u_{jk}).
- **Soft working time constraints**. Employee i can work more or less than the required number of days (a_i), with the penalty u_i and l_i per day respectively.
- **Preference constraints**. This kind of constraints represents the preference of assignment, by setting up requests for assignments with violation penalties. The cost of assignment is still $c_{i,j,k}$, but there is a penalty $d_{i,j,k}$ if employee i is requested to work shift k on day j while such assignment is not in the schedule [4].

To represent these constraints in the min-cost network flow model, we develop a method using dummy nodes and arcs. Problem P_2, which minimizes the sum of costs and penalties of the assignment and considers the assignment constraints, skill constraints, soft staffing constraints, soft working time constraints and preference constraints, is used as an example to present the method.

Figure 2 illustrates the min-cost network flow model with dummy arcs and nodes for P_2. The supplies of dummy source nodes a and m are M and

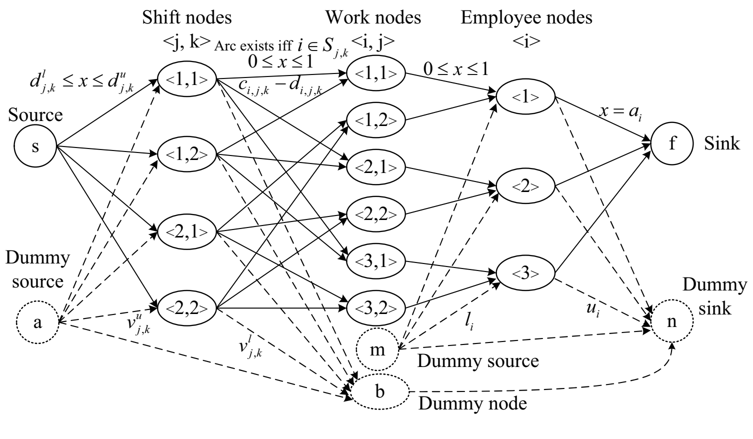

Fig. 2: The min-cost network flow model for problem P_2

M respectively, and the demand of the dummy sink node n is $2M$ (M is a sufficiently big integer, for example, $\sum_{i \in I} a_i$). The supply of source node s is $\sum_{i \in I} a_i$, equaling the demand of sink node f.

The dummy source node a, dummy node b, and dotted dummy arcs related to them are key to representing the soft staffing constraints. If there is no violation of this kind of constraints, all the flows in the dummy source node a are sent directly to the dummy node b without any cost. Otherwise, the over-staffing penalties are calculated by the flows from the dummy source node a to the shift nodes $<j,k>$ with cost $v^u_{j,k}$, and the under-staffing penalties ($v^l_{j,k}$) are represented by the costs of flows from the shift nodes $<j,k>$ to the dummy node b.

The dummy source node m and dummy arcs related to the employee nodes $<i>$ and dummy sink node n are introduced for the soft working time constraints. The excessive working days of employee i are represented by the flows from employee nodes $<i>$ to the dummy sink node n with cost u_i. When the working days (a'_i) of employee i is less than a_i in the schedule, there is an amount of flows ($a_i - a'_i$) sent from the dummy source node m to employee nodes $<i>$ with cost l_i.

For the preference constraints (shift requests), we replace the penalty $d_{i,j,k}$ and cost $c_{i,j,k}$ by a new cost $c_{i,j,k} - d_{i,j,k}$ associated with the arcs from the shift nodes $<j,k>$ to the work nodes $<i,j>$, as the cost of assignments regarding the preference constraints in Equation (1) is equal to (2), in which $d_{i,j,k}$ is a constant and $c_{i,j,k} - d_{i,j,k}$ is incurred by the assignment $f_{i,j,k} \in \{0,1\}$.

$$\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} c_{i,j,k} f_{i,j,k} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} d_{i,j,k}(1 - f_{i,j,k}) \tag{1}$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (c_{i,j,k} - d_{i,j,k}) f_{i,j,k} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} d_{i,j,k} \tag{2}$$

4 Discussion and future work

In this paper, we study the soft constraints in personnel scheduling problems which are important in practice and missing in existing minimum cost network flow models. New network models with dummy nodes and arcs are proposed without losing the polynomially solvable property.

In the future, we will focus on the solution methods for more complicated personnel scheduling problems, based on the polynomially solvable model which is common in many applications and research.

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